VERSION Draft

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STATISTICS

Mathematical Analysis
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B.Sc 3rd semester
(Take Home Assignment 3)
Surprise Test
Send by 19 January, 2022

(Time allowed: 1 **hours)**

NOTE: There are 6 questions. All question carry 5 marks (total 30 marks).

Convergence of Sequence & Series & Miscellaneous

1. State with proof that the following series is convergent/divergent:

$$1 + \frac{3}{2!} + \frac{5}{3!} + \frac{7}{4!} + \cdots .$$

2. Discuss the convergence/divergence of the following series:

$$\sum_{n=1}^{\infty} \frac{n^{n^2}}{(1+n)^{n^2}}.$$

3. Show that, the sequence $a_n = \left(1 + \frac{1}{n}\right)^{(n+1)}$ is strictly monotone decreasing.

4. The sequence a_n is monotone and it has a convergent subsequence. Does it imply that a_n is convergent? State with proof.
(Hint: If subsequence $a_{n_k} \to a$, then from monotonicity, $\forall\ n > n_k,\ |a_n - a| \leq |a_{n_k} - a|$.)

5. Prove that, $\sum_{n=1}^{\infty} \frac{1}{n^2} < 2$.

6. (True/False) If x_n is convergent, y_n is divergent, then $(x_n \cdot y_n)$ is divergent. If true, give reasons. If false, give counterexample to show the statement is false.